COMPARISON OF ANALYTICAL AND NUMERICAL CALCULATION

OF LOAD-BEARING GLASS STRUCTURE

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Abstract

In modern architecture, glass is used more and more frequently not only for fillings of openings but also for load-bearing structural elements. However, currently, there is a lack of mandatory provisions for the design of glass structures. Regarding the European standards, there is a draft of prEN 13474 "*Glass in Building*" where you can find the calculation procedure of the maximum tensile stress and deflection of simply supported rectangular glass panes with uniformly distributed loads. This regulation is used for the following model calculation of the maximum tensile stress in the glass pane of a shelter (roof). The calculation of the shelter is performed in two variants. In the first case, the shelter consists of a single-layer glass, which is placed on a supporting steel structure along three sides of the glass pane. In the second case, a laminated glass is used. The results calculated according to the standard are compared with a numerical model of the structure created in the add-on module RF-GLASS of the program RFEM by Dlubal Software GmbH. Unlike the manual calculation, the advantage of the numerical model is the possibility to consider foil properties of the laminated glass, and thus achieve more accurate results. The entered foil properties play an important role in the redistribution of stresses and deformations, which is obvious from the numerical study of several calculation variants.

Key words: glass, PVB, design, prEN 13474, numerical model

1   Introduction

Regarding the current lack of general binding rules for glass structure designing in the European Union, a designer has to rely on experimentally found rules or numerical calculations. The European Committee for Standardization (CEN/TC129/WG8) has prepared a draft of the European Standard prEN 13474 "*Glass in Buildings*" – Part 1 and Part 2, but the document is accessible only with limits and at present, the completion of it is being processed [1], [2]. Another disadvantage of the currently available regulations of prEN (the content was partially overtaken from the original DIN EN), where the designers can take their inspiration for a load-bearing glass structure design, is the extent of the regulations covering only the plate elements such as facade components, doubled insulating glass or laminated glass (incomplete). The issue of load-bearing glass structures is described more generally in the Australian standard AS 1288 "*Glass in Buildings – Selection and Installation*" from 2006, [3], for instance. For a design of atypical structures of glass, beams, ribs or for stability issues, the scientific publications or experimental research results based on numerical studies can be useful.

2   Description of examined structure

As an example for the calculation, there is a glass pane of a horizontally glazed shelter, supported continuously on three sides by a system of IPE beams and angles connected to the house facade, see *Figure* 1.
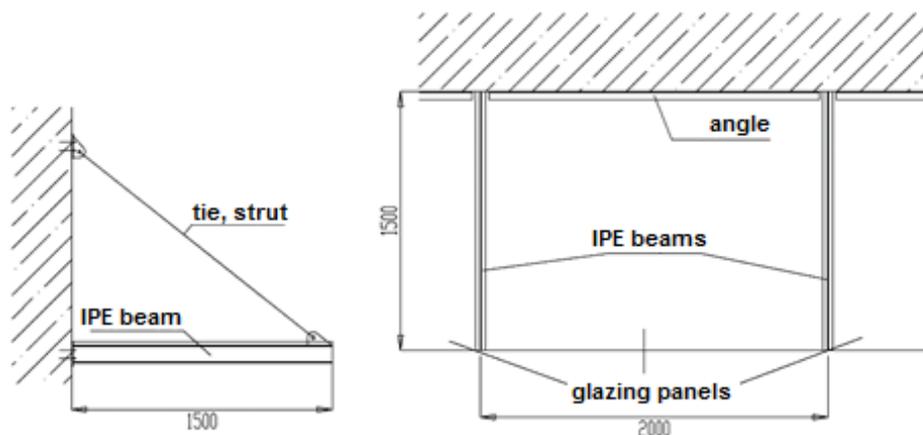

Figure 1: *Scheme of the examined shelter structure*

Glazing panels are placed on a supporting structure by a glued joint that transfers forces from the glass pane to the supporting structure and also prevents effectively from direct contact of

the glass and steel, see *Figure 2*. In this system, the horizontal force of a tie is transferred directly by the steel IPE cross-section, and therefore is not reflected in the calculation of the glass part.
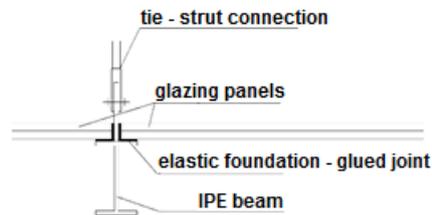

Figure 2: *Detail of the glazing panel foundation on the steel structure*

Design of the steel structure and the glued joint is not a part of this work. This work deals only with the design of the glass pane performed in the variants of a single-layer glass and laminated (two-layer) glass with the PVB foil. The calculation is carried out according to a procedure described in prEN 13474-2 "*Glass in Building - Design of glass panes - Part 2: Design of uniformly distributed loads*" and by means of a numerical model of the structure in the add-on module RF-GLASS of the program RFEM 5 by Dlubal Software. In the conclusion, both calculation methods are compared.

3   Loading

The governing load cases are calculated according to CSN EN 1991-1-3: *Snow load* and CSN EN 1991-1-4: *Wind load*, [4], [5].

The characteristic value of the snow load in Prague is 0.7 kN/m². The shelter will be placed most likely on a facade of an apartment building or another house, so the shape coefficient μ_2 considering the snow drift must be included. The value of the coefficient μ_2 is set to 2.0 and it includes snow drift as well as the snow fall from the roof. Therefore, the characteristic value of the snow load is $s_k = \textbf{1.4 kN/m}^2$**.**

The wind load is determined by a detailed calculation for the wind zone II., terrain category IV. (placed in a street net of a city); the basic dynamic pressure is $q_p(z) = \textbf{0.46 kN/m}^2$**.** The coefficient C_{pe} for the negative wind pressure on the shelter is considered in a simplified way as a weighted average of particular values for the whole shelter roof surface by a constant

value -1.69 (with the fullness coefficient $\varphi = 1.0$). So the value of the surface load by the negative wind pressure is w_k = **-0.78 kN/m².** The load by the positive wind pressure on the shelter has a lower value in comparison with the negative load. For $C_{pe} = 0.85$ is w_k = **0.39 kN/m².**

3.1 Load Combination

1) CO1 – 1.35 x self-weight of the glass pane + 1.5 x snow + 1.5 x $\psi_{0.1}$ (according to prEN 13474-1) x wind positive

2) CO2 - 1.0 x self-weight of the glass pane + 1.5 x wind negative

The governing combination for the glass pane design is CO1 while CO2 is governing for the design of the glued joint stressed by tension under the negative wind pressure. Also, CO2 may be governing for the design of the inclined member with a light structure stressed by compression under this combination, which is crucial for the determination of the member's dimensions. This work deals with the design of the glazing panel; the steel structure and the glued joint are not considered. Therefore, only the combination CO1 is considered for further calculations. According to prEN 13474-1, the combination coefficient $\psi_{0.1}$ for the wind load is taken into account with the value 0.15 in ULS and SLS, [1]. The influence of the positive wind pressure on the surface of the shelter roof is very low in comparison with the snow load.

4 Calculation according to prEN 13474-2 "*Glass in Building - Design of glass panes - Part 2*"

For rectangular glass panes simply supported on three sides (*Figure 3*), the maximum tensile stress in the glass is defined by the relation of (1) and the deformation by (2). The coefficient values k_1, and k_4 (see *Table 1*) are approximate, the verified and exact results will be available after the standard is finished, [2]. Therefore, the numerical calculation is always more precise for the design at the present.
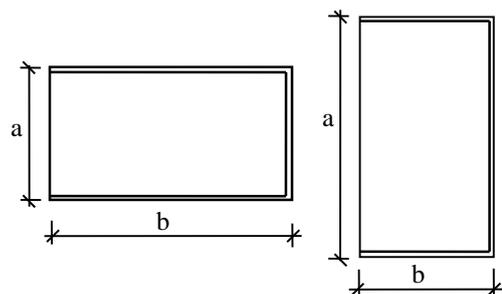

Figure 3: *Labeling of rectangular panes supported on three sides*

$$\sigma_{max} = k_1 \frac{a^2}{h^2} F_d ,$$ (1)

$$w_{max} = k_4 \frac{a^4}{h^3} \frac{F_k}{E}$$ (2)

Table 1: *Coefficients k_j for the stress and deformation calculation of the rectangular pane supported on three sides, [2]*

$\lambda = b / a$	k_1	k_4
0.3	0.16	0.048
0.4	0.26	0.064
0.5	0.36	0.080
2/3	0.45	0.106
1.0	0.67	0.140
2.0	0.79	0.165
4.0	0.80	0.167

4.1 Determining Design Strength of Glass

The characteristic value of the soda-lime-silica and borosilicate glass is defined as $f_{g,k} = 45$ MPa, [1], in compliance with prEN 13474-1. The structure is made of the thermally toughened glass with the characteristic strength $f_{b,k} = 120$ MPa. The design strength of the thermally toughened glass is defined according to the relation (3):

$$f_{g,d} = \left(\frac{f_{b,k} - f_{g,k}}{\gamma_V} + k_{mod} \frac{f_{g,k}}{\gamma_m k_A} \right) \gamma_n ,$$ (3)

where γ_n is the national partial factor defined by national customs (recommended value is 1.0),

k_A is the size factor defined by the size of the glass pane A [m^2] according to the relation $k_A = A^{0.04}$,

k_{mod} is the modification factor that reflects the main stress duration effect, ($k_{mod} = 0.36$ according to prEN 13474-1 for dominant snow load),

γ_m is the material safety factor of float glass, ($\gamma_m = 1.8$ according to prEN 13474-1),

γ_v is the material safety factor of thermally toughened glass ($\gamma_v = 2.3$ according to prEN 13474-1), [1].

By inserting the values in the formula (3), the following result is obtained:

$$f_{g,d} = \left(\frac{120 - 45}{2.3} + 0.36 \frac{45}{1.8 \cdot (1.5 \cdot 2.0)^{0.04}}\right) \cdot 1.0 = 41.2 \text{ MPa}.$$

4.2   Calculation of Single-Layer Glass with Thickness of 19 mm

For the glass pane design, CO1 is governing, where:

self-weight load $G_k = 0.019 \cdot 25 = 0.48 \text{ kN/m}^2$,

snow load $Q_k = 1.4 \text{ kN/m}^2$,

positive wind pressure load $W_k = 0.39 \cdot 0.15 = 0.06 \text{ kN/m}^2$.

$F_k = 0.48 + 1.4 + 0.06 = 1.94 \text{ kN/m}^2$,

$F_d = 0.48 \cdot 1.35 + (1.4 + 0.06) \cdot 1.5 = 2.84 \text{ kN/m}^2$,

$$\lambda = \frac{b}{a} = \frac{1500}{2000} = 0.75,$$

$k_1 = 0.50$ (see *Table 1*), satisfied.

$$\sigma_{max} = k_1 \frac{a^2}{h^2} F_d = 0.50 \cdot \frac{2000^2}{19^2} \cdot 2.84 \cdot 10^{-3} = 15.7 \text{ MPa} \leq 41.2 \text{ MPa},$$

$k_4 = 0.114$ (see *Table 1*), satisfied.

$$w_{max} = k_4 \frac{a^4}{h^3} \frac{F_k}{E} = 0.114 \cdot \frac{2000^4}{19^3} \cdot \frac{1.94 \cdot 10^{-3}}{70000} = 7.4 \text{ mm} \leq \frac{L}{250} = \frac{2000}{250} = 8 \text{ mm},$$

Since the single-layer thermally toughened glass shows no residual resistance after the failure, it is not eligible to be used in the examined structure. The calculation in this chapter is only an example of how to use the procedure of prEN 13474-2 and is generally applicable also to

structures of another type, [2]. In the following chapter, there is a calculation of the laminated safety glass that is usually used for this kind of structures.

4.3 Calculation of Laminated Glass (Thickness 2 x 15 mm) and Intermediate PVB Layer (4 x 0.38 mm)

Appendix F, prEN 13474-2, presents values for the shear force transfer factor Γ, which is crucial for the effective thickness of laminated glasses that is possible to consider in the calculation, see *Table 2*, [2]. This table is currently being extended to cover the intermediate values for various materials of intermediate layers, sorted in groups by stiffness. The shear stiffness of the intermediate layer G significantly affects the interaction of both panes and thus the stress condition and deformation of the entire structure. Besides the material of the intermediate layer, the shear interaction depends on the load duration and on the temperature. However, it is safe to omit the shear interaction of both panes during the manual calculation of snow load on side of the laminated glass with PVB foil.

Table 2: *Shear force transfer factor for laminated glass, [2]*

Glass type	Shear force transfer factor Γ	
	Short-term load, e.g. wind	Other load
Laminated glass	0	0
Laminated safety glass	1	0

For the factor $\Gamma = 0$, the effective thickness for the deformation calculation is defined according to the relation (4)

$$h_{ef,w} = \sqrt[3]{\sum_i h_i^3}$$

and for the stress calculation of the (j-th) glass pane according to [2]. (5)

$$h_{ef,\sigma,j} = \sqrt{\frac{\sum_i h_i^3}{h_j}},$$

For the glass pane design, CO1 is governing, where:

self-weight load $G_k = 0.030 \cdot 25 = 0.75$ kN/m^2,

snow load $Q_k = 1.4$ kN/m^2,

positive wind pressure load $W_k = 0.39 \cdot 0.15 = 0.06$ kN/m^2.

$F_k = 0.75 + 1.4 + 0.06 = 2.21$ kN/m^2,

$F_d = 0.75 \cdot 1.35 + (1.4 + 0.06) \cdot 1.5 = 3.20$ kN/m^2,

by substitution for (4) $h_{ef,w} = \sqrt[3]{(15^3 + 15^3)} = 18.9$ mm,

by substitution for (5) $h_{ef,\sigma,j} = \sqrt{\dfrac{(15^3 + 15^3)}{15}} = 21.2$ mm ,

$\lambda = \dfrac{b}{a} = \dfrac{1500}{2000} = 0.75$,

$k_1 = 0.50$ (see *Table 1*), satisfied.

$\sigma_{max} = k_1 \dfrac{a^2}{h_{ef,\sigma,j}^2} F_d = 0.50 \cdot \dfrac{2000^2}{21.2^2} \cdot 3.20 \cdot 10^{-3} = 14.3$ MPa ≤ 41.2 MPa,

$k_4 = 0.114$ (see *Table 1*),

$w_{max} = k_4 \dfrac{a^4}{h_{ef,w}^3} \dfrac{F_k}{E} = 0.114 \cdot \dfrac{2000^4}{18.9^3} \cdot \dfrac{2.21 \cdot 10^{-3}}{70000} = 8.5\ mm \geq \dfrac{L}{250} = \dfrac{2000}{250} = 8$ mm.

The requirement for the pane deflection set in prEN 13474-2 is not satisfied. Since this is the conservative manual calculation of the deflection, we can suppose a certain reserve. More exact calculation using the Finite Element Method will prove the serviceability of the designed structure. In addition, the standard does not mention any special requirements for limit deformation of glass panes; these are defined considering the serviceability of the structure or considering the connection of the entire system.

5   Calculation in FEM Software

The numerical analysis is carried out in the RFEM program with the RF-GLASS add-on module by Dlubal Software GmbH, which allows the calculation of the single-layer and laminated glass as well as the insulating glass. Similarly to the manual calculation, both varieties were calculated to enable the comparison of both types of calculations. In the laminated glass model, it is possible to predefine the shear interaction of the PVB foil (and other intermediate layers) with regard to the load duration or the environment temperature. This can be done by various shear stiffness values of the intermediate layers corresponding to the relevant conditions, to which the glass element will be exposed. These experimentally found values are possible to obtain directly from manufacturers of the intermediate layers. The RF-GLASS add-on module of the RFEM 5 program allows the definition of no shear coupling of the individual glass panes, which is the case of the manual calculation.

5.1   Calculation of Single-Layer Glass with Thickness of 19 mm

Similarly to the manual calculation, there are two load combinations also for the numerical calculation. One combination covers the ultimate limit state with the design load values (CO1). The second combination covers the serviceability limit state with the characteristic load values (CO2). Self-weight, snow load and wind load were considered, see previous chapters.

CO1: $0.48 \cdot 1.35 + (1.4 + 0.06) \cdot 1.5 = 2.84 \ \text{kN/m}^2$

CO2: $0.48 + 1.4 + 0.06 = 1.94 \ \text{kN/m}^2$

The calculation is based on the Mindlin plate theory. The size of the finite element mesh is 50 x 50 mm. The main design tensile stress is 17.1 MPa in this case, see *Figure 4*.

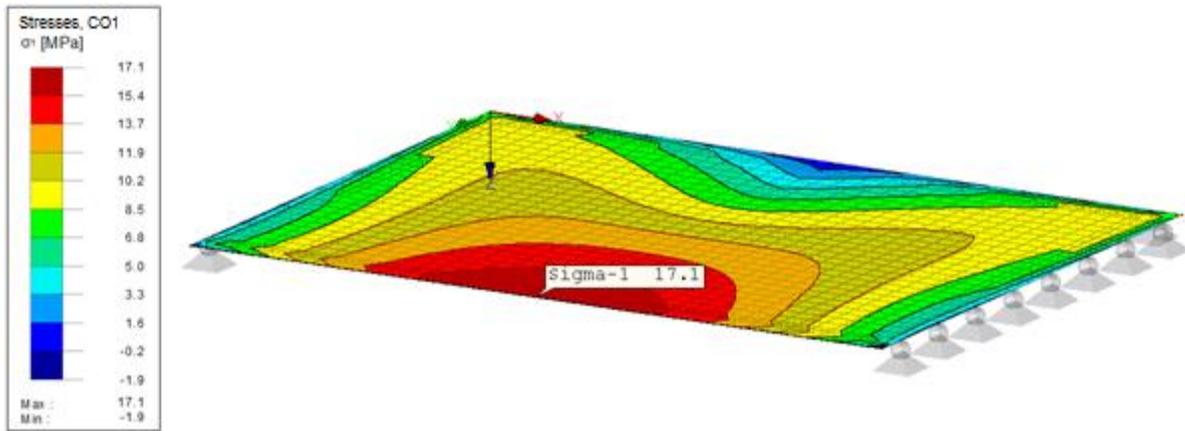$\sigma_{max} = 17.1 \ \text{MPa} \leq 41.2 \ \text{MPa}$, satisfied.

Figure 4: *Stress diagram σ_1 on the bottom layer of the single-layer glass for CO1 in RF-GLASS*

Deflection of the single-layer glass is pictured in *Figure 5*. In the middle of the non-supported edge, the variable load deflection is 5.6 mm. The permanent load deflection in the same location is 1.8 mm. So the total deflection calculated on the numerical model is 7.4 mm.
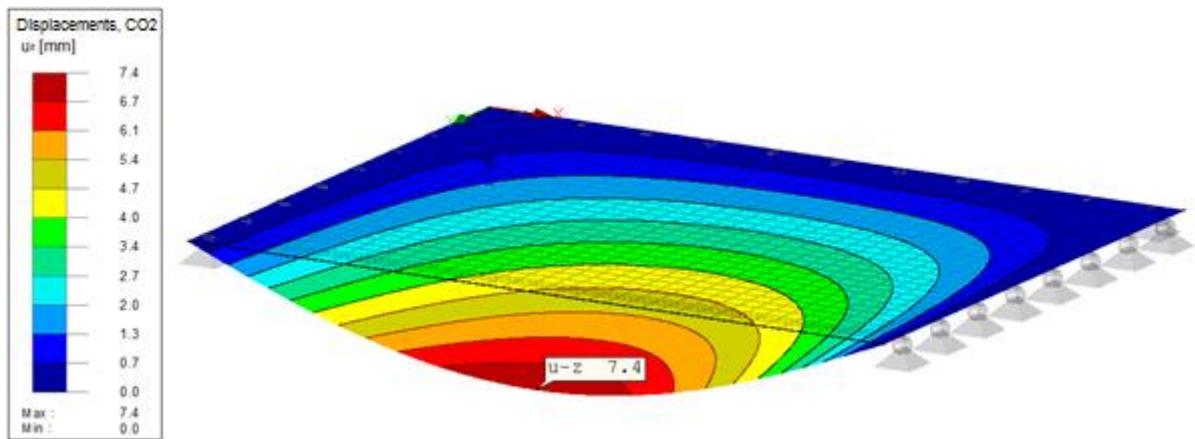

Figure 5: *Single-layer glass deflection for CO2 in RF-GLASS*

5.2   Calculation of Laminated Glass with Thickness of 2x15 mm

For the calculation of the laminated glass, the same load combinations were used as for the manual calculation. CO1 stands for the ultimate limit state with design load values, CO2 for the serviceability limit state with characteristic load values.

CO1: $0.75 \cdot 1.35 + (1.4 + 0.06) \cdot 1.5 = 3.20 \text{ kN/m}^2$

CO2: $0.75 + 1.4 + 0.06 = 2.21 \text{ kN/m}^2$

The laminated glass consists of two glass panes with a thickness of 15 mm and the intermediate PVB layer with a thickness of 1.52 mm (4x0.38 mm).

Since the RF-GLASS module allows the calculation of laminated glasses according to the plate theory or by precise modeling using solids, we create several models of the laminated glass and compare the results of these models. In all cases, the size of the finite element mesh is set to 50 x 50 mm.

5.2.1 Calculation using Plate Theory

During the manual calculation, the shear interaction of both panes was entirely omitted. Therefore, the numerical model is calculated without the shear coupling of layers as well.

The main design tensile stress of the glass is 15.5 MPa, see *Figure 6*.

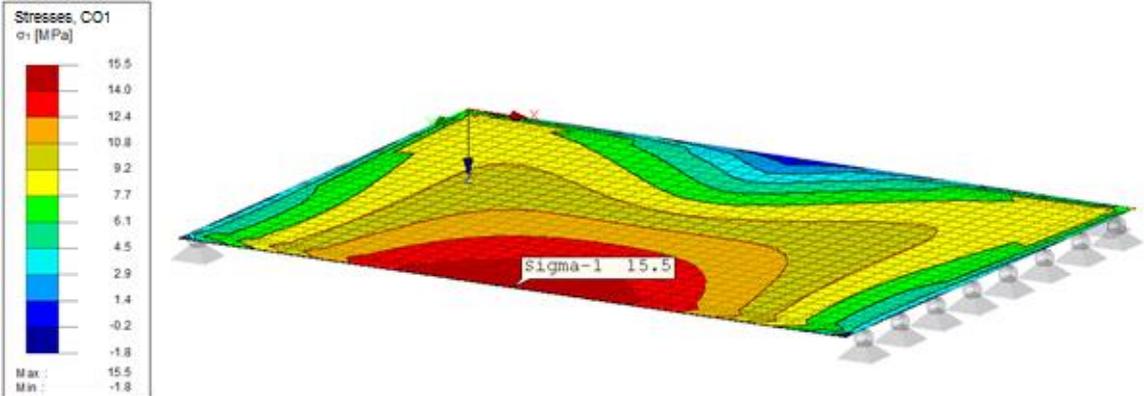$\sigma_{max} = 15.5$ MPa ≤ 41.2 MPa, satisfied.

Figure 6: *Stress diagram σ₁ on the bottom layer of the laminated glass for CO1 in RF-GLASS*

Deflection of the laminated glass is pictured in *Figure 7*. In the middle of the non-supported edge, the variable load deflection is 5.7 mm. The permanent load deflection in the same location is 2.9 mm. So the total deflection of the numerical model is 8.6 mm.
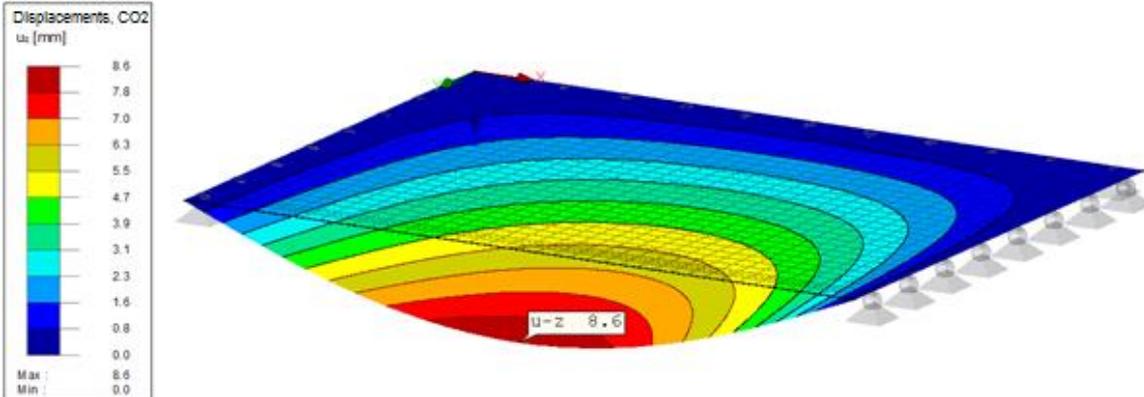

Figure 7: *Laminated glass deflection for CO2 in RF-GLASS*

5.2.2 Calculation using Solids

In RF-GLASS, it is possible to perform the 3D calculation (modeling using solids). In this calculation, the PVB foil properties are very important, so there are two calculations with different foil properties performed for comparison purposes. In both cases, the line supports are located on the lower edge of the glass.

5.2.2.1 Variant A

In the first variant, the following PVB foil properties of the long-term load are taken into account: $E = 0.03$ MPa, $G = 0.01$ MPa.

The main design tensile stress in the location of the greatest deflection is

$$\sigma_{max} = 10.7 \text{ MPa} \leq 41.2 \text{ MPa, therefore satisfied.}$$

In the middle of the non-supported edge, the variable load deflection of the glass pane is 3.3 mm. The permanent load deflection in the same location is 1.7 mm. So the total deflection of the numerical model is 5.0 mm.

5.2.2.2 Variant B

In the second variant, different foil properties are considered to demonstrate the extent of the effect on the results. The following properties are taken into account:

$E = 3.00$ MPa, $G = 1.00$ MPa.

The main design tensile stress in the location of the greatest deflection is

$$\sigma_{max} = 5.0 \text{ MPa} \leq 41.2 \text{ MPa, therefore satisfied.}$$

The total deflection is 2.7 mm.

6   Results Comparison

The comparison of the results of the manual calculation and the numerical study (see *Table 3*) shows that the calculation according to prEN 13474-2 is relatively accurate for a single-layer glass but it does not always need to be too conservative. Regarding the high material safety factors for the glass, the differences of the calculated maximum design stress do not seem to be problematic. However, it is recommended - especially for deflection calculations - to assume that the values of the analytical calculation will be reached, and thus keep some reserve in this kind of structural design.

Table 3: *Comparison of the results calculated according to prEN 13474-2 and the numerical study in RF-GLASS*

Glass type	Calculation type	Results	
		Design stress [MPa]	Deflection [mm]
Single-layer glass	prEN 13474-2	15.7	7.4
	Numerical calculation Plate theory	17.1	7.4
Two-layer glass with PVB foil	prEN 13474-2	14.3	8.5
	Numerical calculation Plate theory	15.5	8.6
	Numerical calculation 3D, Variant A	10.7	5.0
	Numerical calculation 3D, Variant B	5.0	2.7

The laminated glass calculation according to prEN 1347-2 is conservative because it does not consider any shear interaction of both panes (similarly to the model calculated by means of the plate theory without shear coupling of layers). Therefore, the values of the maximum tensile stresses and deflections are higher than the values of the 3D calculation. The values in *Table 3* show that even a small effect of the shear stiffness on a long-term loaded PVB foil and the precise numerical calculation using solids may reduce the stresses and deformations of the manual calculation, as shown in *Table 3*. The table also shows that PVB foil properties affect the results significantly. So it is always necessary to find out the required properties directly from a producer.

7 Conclusion

The manual calculation according to European standard draft prEN 13474-2 can be used for the design of glass pane structures, but the calculation should be verified by another tool (a numerical analysis or an experiment). *Table 1* providing the important factors for the calculation currently includes only approximate values, as the standard authors admit. In the case of the manual calculation of a laminated glass, it is currently not possible to consider any shear interaction of individual glass panes for any but short-term load (unlike the numerical calculation). Therefore, the numerical calculation represents a powerful tool for safe and economical design of a load-bearing glass pane, especially for the laminated glass.

Literature

[1] prEN 13474-1. *Glass in Building - Design of glass panes - Part 1: General basis of design*. January 1999.

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[5] CSN EN 1991-1-4 (730035). *Eurocode 1: Actions on structures - Part 1-4: General actions - Wind loads*. April 2007.